MODES IN A MÖBIUS WIRE-LOADED CAVITY RESONATOR

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ABSTRACT: *Resonant modes of Möbius wire-loaded cavities are determined by a fully three-dimensional numerical eigenfrequency analysis. Excellent agreement is achieved with experimental measurements of a loosely coupled Möbius wire in a copper cavity, for both the Möbius modes and the conventional generalized even modes. The field solutions provide insight applicable to suppression of the higher order modes that limit the out-of-band performance of the bandpass filters implemented with Möbius resonators.* © 2001 John Wiley & Sons, Inc. Microwave Opt Technol Lett 31: 6–9, 2001.*

Key words: *microwave resonators; topology; electromagnetic modeling*

1. INTRODUCTION

Dual-mode Möbius resonators which rely on a geometrical deformation of a transmission line to obtain a fourfold reduction in volume have been previously introduced [1, 2]. These resonators are related to the study of a nonorientable surface in the mathematical language of topology. Although traditionally referred to as one-sided surfaces, nonorientable surfaces are those for which the concept of left and right are globally nonsensical [3].

Since left and right are not defined globally on a nonorientable surface, there appears to be a periodic alternation between left and right as the center circle of a Möbius strip is traversed. Similarly, a traveling wave on a transmission line experiences periodic reversals in polarity. By projecting a transmission line onto a nonorientable surface and phasing the electromagnetic oscillation with the path length associated with reversal of left and right, a resonant condition has been shown to occur at a wavelength twice that of an equivalent transmission line on a traditional orientable surface. For an equivalent resonant frequency, this corresponds to a factor of 4 reduction in resonator volume [1, 2].

An alternative way of visualizing the dominant mode is to consider that, since the Möbius strip contains a 180° twist, the distributed electromagnetic analog is expected to require a "circumference" corresponding to a half wavelength. When combined with the 180° twist, a total phase change of 360° occurs, and a resonant condition results.

The above description of the resonant condition for a Möbius resonator applies well to the fundamental mode, and a detailed explanations of the dual-mode nature of this resonance have been provided elsewhere [1, 2]. However, an understanding of the higher order Möbius modes as well as other modes can be greatly facilitated by the use of an accurate technique for computing the electromagnetic field patterns of the modes. In subsequent sections, we will compare the measured and computed resonant frequencies for several modes, and examine the field distributions. This accurate computational approach is being pursued since the insight provided is proving useful in developing techniques for suppressing out-of-band resonances in Möbius bandpass filters.

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2. MEASUREMENTS

A cylindrical copper cavity with a diameter of 2.5 cm and a height of 0.9 cm was used for these measurements. When empty, this cavity has a fundamental frequency at 9.04 GHz. The cavity is closed with a single end plate that is clamped with four screws. Input and output ports were placed 180° apart in the cylindrical wall. Small coupling loop antennas, fabricated from 0.085 in outer diameter coaxial cable, were used to examine the properties of the resonances. The coupling strength could be increased by sliding the loop further into the cavity.

The Möbius wire structure was fabricated from 0.085 in diameter coaxial cable from which the outer conductor had been removed, leaving the center conductor sheathed in the dielectric. The Möbius wire structure was hand-shaped from a single piece of 12.2-cm-long cable so that the mean diameter was 1.94 cm. Four small glass-reinforced dielectric spacers were used to maintain a uniform wire separation.

Figure 1 shows the Möbius wire resonator placed in the copper cavity. The measured response is shown in Figure 2, where three distinct pairs of resonances are evident. The pairs of resonances at approximately 2 and 6 GHz are the two orthogonal fundamental and two orthogonal second-order Möbius modes. As discussed below, the pair of resonances near 4 GHz are not Möbius-type modes. Coupling to these modes is the largest hindrance in realizing a good out-of-band response in Möbius bandpass filters. Electromagnetic modeling of this structure yields the origin of these modes, and hence, insight into methods for their suppression.

3. NUMERICAL CALCULATIONS

The resonator was modeled numerically using CTLSS [4], a fully 3-D, general-geometry, frequency-domain electromagnetics code for determining resonant modes in complex cavities. It uses the Jacobi–Davidson algorithm to selectively determine all cavity eigenmodes in a specified frequency range, including degenerate modes.

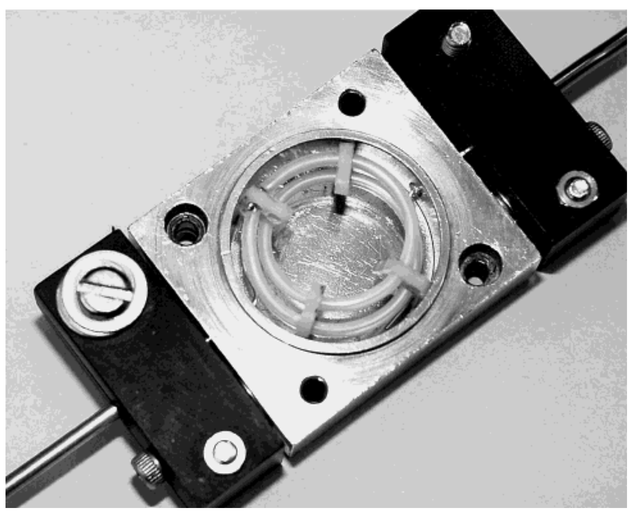


Figure 1 Photograph of a Möbius wire resonator in a 1 in diameter cylindrical copper cavity with the lid removed

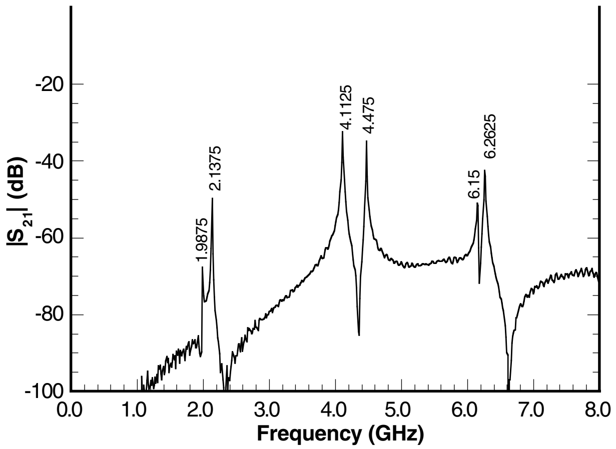


Figure 2 Measured response of the loosely coupled Möbius wire-loaded cavity shown in Figure 1. The measured resonant frequencies are clearly annotated

The Möbius wire was modeled as the union of two helical wire segments, enveloped by a dielectric sheath ($\varepsilon = 2.05$) to represent the insulation, and placed inside a perfectly conducting pillbox cavity. All dimensions were matched to the experimental device.

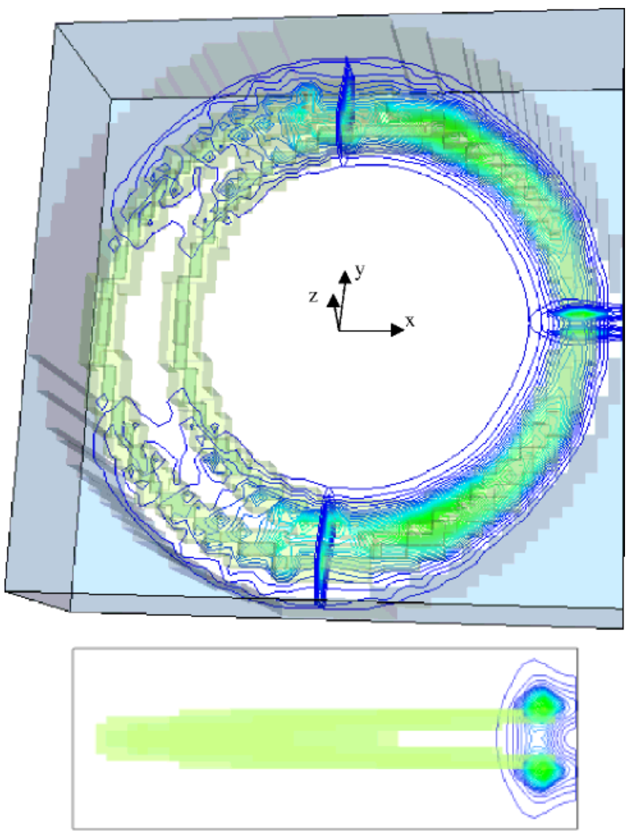
All eigenmodes in the frequency range 1–10 GHz were solved, obtaining both the eigenfrequencies and the corresponding three-dimensional electric field vectors throughout the cavity. The computation of nine eigenmodes took 106 min on a 600 MHz PC.

The discretized field solutions for this device can be visualized by plotting contours of constant electrical energy density on cross-sectional surfaces. Figures 3–8 show the discretized field energy distribution for the first six eigenmodes.

4. DISCUSSION

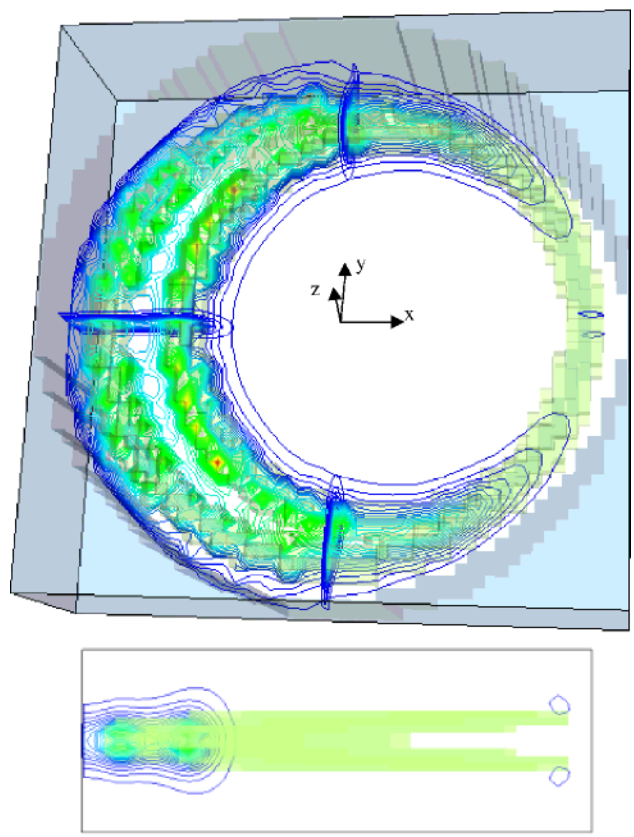
Figures 3 and 4 show the two orthogonal fundamental Möbius modes. They are characterized by each having a single energy maximum circumferentially. It is evident that the energy is concentrated between the wires for both of these modes, and that the effects of the cavity walls are secondary. As seen by the electric-field intensity plots, splitting of the modes occurs such that the electric-field strength is maximum where the wires are at the same radius in Figure 3 ($\theta = 0°$) and where the wires are at equal elevation in Figure 4 ($\theta = 180°$). This is consistent with the local cross-sectional geometry for the transmission line experiencing two extremes in parasitic capacitance with respect to the cavity walls. When examining these modes, it is useful to consider the local cross section where these modes act as generalized odd modes with respect to the cavity wall.

Figures 5 and 6 show the next pair of resonant modes from Figure 2. Unlike the modes shown in Figures 3 and 4, a significant fraction of the field energy is concentrated between the wires and the cavity wall. Indeed, a Möbius mode



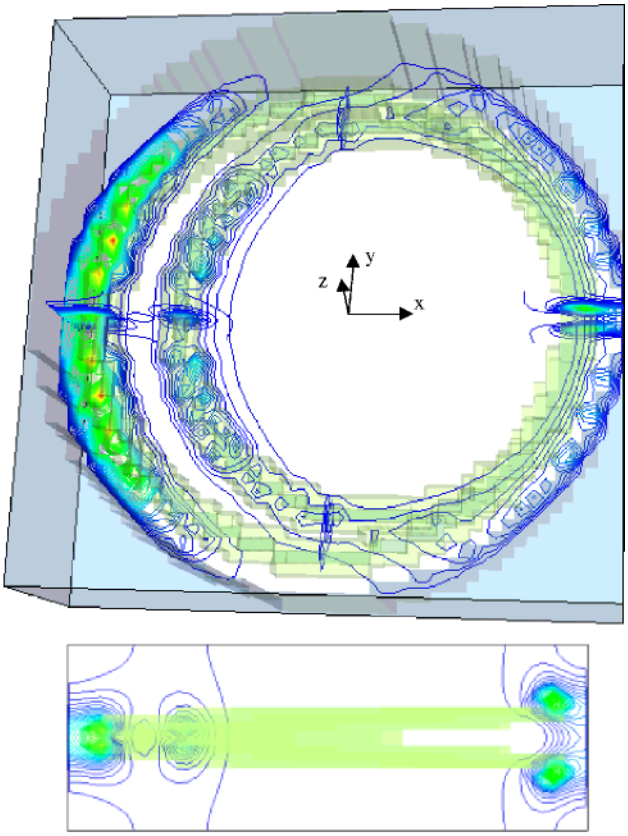
Electric field in the x-z plane at y = 0.

Figure 3 Field distribution for the resonant mode calculated at 2.016 GHz which corresponds to the measured value of 1.9875 GHz [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]



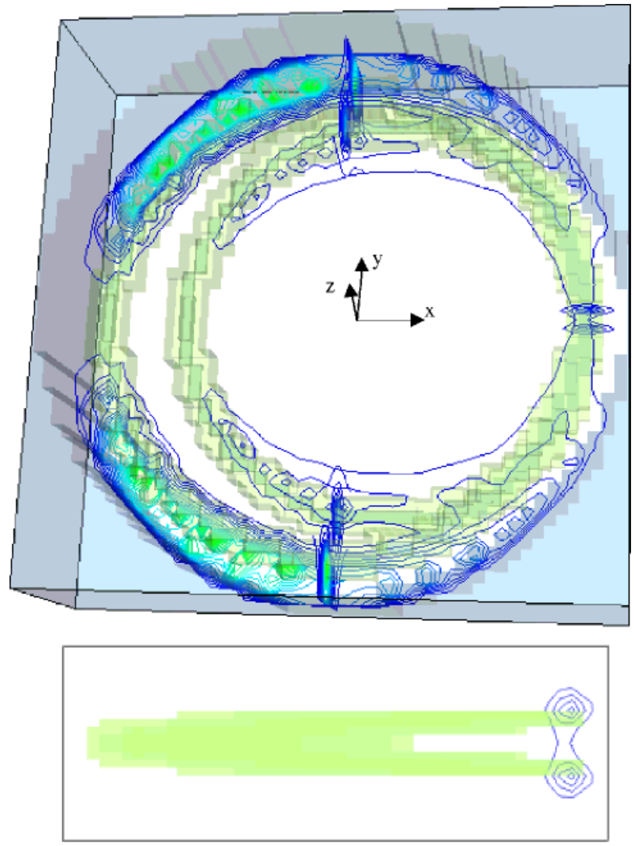
Electric field in the x-z plane at y = 0.

Figure 4 Field distribution for the resonant mode calculated at 2.101 GHz which corresponds to the measured value of 2.1375 GHz [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]



Electric field in the x-z plane at y = 0.

Figure 5 Field distribution for the resonant mode calculated at 4.261 GHz which corresponds to the measured value of 4.1125 GHz [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]



Electric field in the x-z plane at y = 0.

Figure 6 Field distribution for the resonant mode calculated at 4.455 GHz which corresponds to the measured value of 4.475 GHz [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

cannot exist at this frequency since the circumference of the wire is approximately a full wavelength, and when combined with the 180° twist, an antiresonant condition exists. Hence, these modes are analogous to the fundamental generalized even-type modes [5] with respect to the cavity wall. In Figure 5, the two electric-field maxima are located at the left and right sides ($\theta = 90°$ and $\theta = 270°$), whereas in Figure 6, the electric-field maxima are located at the top and bottom ($\theta = 0°$ and $\theta = 180°$). These modes will occur when $\pi d_e = n\lambda$ (with $n = 1, 2, 3 \ldots$), where πd_e is the mean circumference of the transmission line defining this mode. The frequency splitting of these two orthogonal modes is greater than for the Möbius modes due to the stronger interaction with the cavity wall.

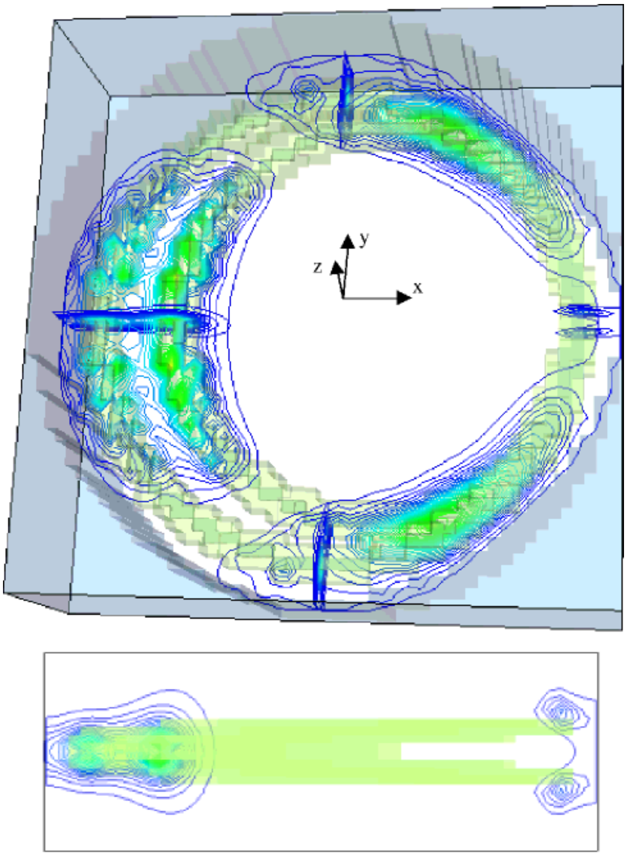
Figures 7 and 8 show the two orthogonal second-order Möbius modes, where it can again be seen that the energy is concentrated between the wires. Also consistent with the behavior of the fundamental modes is that the splitting occurs such that one of the electric-field maxima is located where the wires are at the same radius in Figure 7 ($\theta = 180°$), and where the wires are at equal elevation in Figure 8 ($\theta = 0°$). At this frequency, the circumference of the transmission line corresponds to three halves of a wavelength such that, when combined with the twist in the wires, a resonant condition for a Möbius mode exists. Generalizing, a pair of orthogonal Möbius resonances will exist when $\pi d_o = (2n + 1)\lambda/2$ (with $n = 0, 1, 2 \ldots$), where d_o is the mean circumference of the Möbius resonator.

The agreement between the measured and modeled values for the resonant frequencies is excellent, as can be seen in Table 1. Differences between the actual and modeled frequency values are likely due to a combination of imprecise location of the Möbius wire structure in the cavity and the presence of the dielectric spacers employed to preserve the wire separation, which were ignored in the model.

5. CONCLUSION

Excellent agreement has been achieved between measured and numerically predicted locations of the resonances of Möbius wire-loaded cavity resonators. The Möbius modes, as well as undesired generalized even modes, are accurately predicted by the numerical model, which provides, in addition, a complete description of the electromagnetic fields that cannot easily be obtained by experimental means. The accuracy of this technique and the ability to compute the fields precisely will be invaluable as a design tool in realizing high-performance filters using Möbius wire-loaded cavity resonators.

Future numerical efforts will be focused on determining cavity designs capable of suppressing the generalized even-mode resonances. Capacitive coupling to the fundamental generalized even mode degrades the rejection above the passband of a Möbius bandpass filter. This type of electromagnetic design tool will facilitate the design of segmented

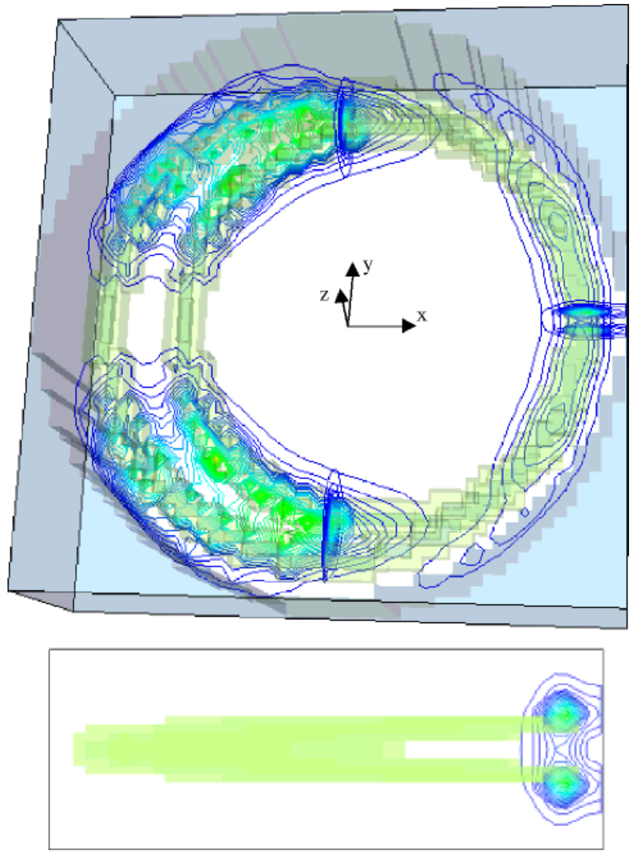


Electric field in the x-z plane at y = 0.

Figure 7 Field distribution for the resonant mode calculated at 6.186 GHz which corresponds to the measured value of 6.15 GHz [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

TABLE 1 Comparison of Measured and Modeled Results

Resonances	Measured (GHz)	Modeled (GHz)	% Error
Fundamental Möbius	1.9875	2.016	1.4
	2.1375	2.101	−1.7
Fundamental "even"	4.1125	4.261	3.6
	4.455	4.475	0.4
Second-order Möbius	6.15	6.186	0.6
	6.2625	6.269	0.1



Electric field in the x-z plane at y = 0.

Figure 8 Field distribution for the resonant mode calculated at 6.269 GHz which corresponds to the measured value of 6.2625 GHz [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

cavities, as well as assist in the optimization of input and output coupling approaches.

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