Letters

Comments on "On the Application of Complex Resistive Boundary Conditions to Model Transmission Lines Consisting of Very Thin Superconductors"

Smain Amari

In the above paper,[1] the authors present an elegant method to account for metallic losses in thin superconductors. The extensive numerical effort, which would otherwise be necessary to exactly solve the boundary-value problem, is considerably reduced by using the concept of complex resistive boundary conditions. This idea seems to have been originally introduced by Senior [1] as the authors point out.

In Section II, the authors introduce a sheet resistance R ((11))

$$R = \frac{1}{\sigma t} \tag{1}$$

where σ is the conductivity of the sheet and t its thickness. The assumption is then made that ((12))

$$\lim_{\substack{t \to 0 \\ \sigma \to \infty}} \frac{1}{\sigma t} = R. \tag{2}$$

What is the physical phenomenon responsible for the increase in the conductivity σ?

In Section III, the authors apply the method to analyze a lossy microstrip line through Galerkin's method. There seems to be, however, a crucial point that may invalidate their numerical results. Indeed, (44) and (45) contain **constant** diagonal elements in addition to the usual components of the lossless Green's impedance dyadics. The authors then expand the current density in a set of basis functions, each of which satisfies the **edge** condition ((46), (47)). Galerkin's method is then applied to determine the propagation properties of the structure.

The presence of the edge condition along with the constant term R in the diagonal elements of the Green's functions leads to integrals of the form

$$R \int_{-w/2}^{w/2} J_{0z}(y) J_{0z}(y)\, dy \propto \int_{-1}^{1} \frac{dy}{1 - y^2}, \tag{3}$$

which are infinite. Parseval's relation was used to calculate the above integral in ordinary space instead of the spectral domain. Also, only the lowest term J_{0z} was considered, but other ones are singular as well. Taking this observation into account, could the authors explain how they obtained numerical results that agree with the analytical solution to the parallel-plate transmission line?

In addition, and taking into account the divergency of the matrix elements, why isn't the attenuation constant infinite since it measures the losses in the structure which are given by (3):

$$\frac{1}{2} \int R J_s J_s^* \, ds \tag{4}$$

where J_s is the surface current? The integral in (4) is singular yet the authors present finite values for the attenuation constant (Fig. 9).

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[1] J. M. Pond, C. M. Krowne, and W. C. Carter, *IEEE Trans. Microwave Theory Tech.*, vol. 37, no. 1, pp. 181–190, Jan. 1989.

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Reply to Comments on "On the Application of Complex Resistive Boundary Conditions to Model Transmission Lines Consisting of Very Thin Superconductors"

J. M. Pond and C. M. Krowne

The comments indicate a misunderstanding of several issues that are central to the above paper.[1] This response will address all three of those issues in the same sequence as they were raised. The first question concerns the resistive boundary condition. The remaining two issues concern the use of the spectral domain approach with the resistive boundary condition to calculate the propagation constant and loss of transmission line structures.

With respect to the first issue, there exists some confusion in the comments concerning an approximation, which is made to reduce the difficulty of the electromagnetic calculation, with a real physical process. By mathematically preserving the value of the sheet resistance, R, in the limit as the sheet thickness is mathematically reduced to zero, a three-region problem is reduced to a two-region problem with a modified boundary condition. As is pointed out in the above paper, the sheet resistance for a superconducting film which is thin compared to the superconducting penetration depth can be described by

$$\frac{1}{\sigma t} = R \tag{1}$$

where σ is the finite complex conductivity of the thin superconductor and t is the finite thickness thereof. In principle, the electromagnetic problem to be solved for the case of an infinite sheet is a three region problem, with region I comprising the region above the sheet, region II being the finite thickness sheet itself, and region III representing the region below the sheet. The electromagnetic

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solution to such a problem can be found by solving for the fields in all three regions, consistent with the boundary conditions at the two interfaces. In certain cases where region II is electrically thin, the three-region problem can be reduced to the problem of solving for the fields in two regions (I and III), subject to the mentioned modified boundary condition which describes the thin sheet region. For a superconductor, electrically thin means much thinner than the superconducting penetration depth. Just as there is no physical phenomenon responsible for the conductivity of the sheet approaching infinity, there is also no physical basis for the sheet thickness to approach zero. However, if

$$\lim_{\substack{t\to 0\\ \sigma\to\infty}} \left(\frac{1}{\sigma t}\right) = R \tag{2}$$

then the effect of the sheet region on the fields in regions I and III will remain properly defined and the additional complication of solving for the fields internal to the superconductor is avoided. This approximation, in conjunction with (13) and (14) of the above paper, is the boundary condition known as the resistive boundary condition [1].

The remaining concerns pertain to the calculation of losses, propagation constant, and current distributions. The method of solution presented in the above paper is successful because, in the spectral domain, the equations can be algebraically manipulated to specifically avoid the difficulty highlighted by (3) of the comments. In the above paper, the use of the resistive boundary condition together with the spectral domain technique allows the usual components of the impedance dyadic to be modified by a term, given by (2), which describes the thin superconducting film. Specifically, the modification, given by (44) and (45) of the above paper, is to subtract a complex term from the diagonal element of the impedance dyadic. This becomes more obvious, if (44) and (45) of the above paper are restated as

$$\tilde{Z}'_{yy}(\zeta)\tilde{J}_{sy}(\zeta) + \tilde{Z}'_{yz}(\zeta)\tilde{J}_{sz}(\zeta) = \tilde{E}^{e}_{y}(\zeta) \tag{3}$$

$$\tilde{Z}'_{zy}(\zeta)\tilde{J}_{sy}(\zeta) + \tilde{Z}'_{zz}(\zeta)\tilde{J}_{sz}(\zeta) = \tilde{E}^{e}_{z}(\zeta) \tag{4}$$

where

$$\tilde{Z}'_{yy}(\zeta) = \tilde{Z}_{yy}(\zeta) - R \tag{5a}$$

$$\tilde{Z}'_{yz}(\zeta) = \tilde{Z}_{yz}(\zeta) \tag{5b}$$

$$\tilde{Z}'_{zy}(\zeta) = \tilde{Z}_{zy}(\zeta) \tag{5c}$$

$$\tilde{Z}'_{zz}(\zeta) = \tilde{Z}_{zz}(\zeta) - R. \tag{5d}$$

Following a standard spectral domain approach [2], the application of a Galerkin-type method results in the right-hand sides of (3) and (4) being zero by virtue of Parseval's theorem. In the dependent equations, the contribution of R, representing the superconductor, was moved to the left-hand side, since, in the spectral domain, this is a simple algebraic manipulation as was shown in the above paper. When the currents are expanded into a set of basis functions with unknown coefficients, the result is a homogeneous system of equations for which the only nontrivial solution occurs when the determinant is zero. The root of this determinant is the complex propagation constant and is a function of R. A specific advantage of the approach presented in the above paper is the incorporation of a descriptor for the superconducting region into the determinantal equation. Hence, the issue raised in the comments is neither relevant to the method developed in the above paper, nor to the types of problems the method was meant to address.

In a similar manner, concern over the singularity of the integral in (4) of the comments is irrelevant because the method used in the above paper employs a different approach to determine the losses. The losses are given by the complex propagation constant that is the root of the determinantal equation, as is discussed above and as was stated at the end of Section IV of the above paper. Since R describes both the stored energy and dissipated energy in the superconducting film, following the same argument presented in the comments would result in infinite internal energy storage in the superconductor yielding a zero phase velocity. Obviously, such is not the case as excellent agreement with experimental and analytical results was shown in the above paper. The difficulties in attempting to compute the losses via the process of (4) of the comments have been well articulated in the literature [3]–[5]. In the above paper, a set of current basis functions was employed, each of which satisfy the edge condition for an infinitesimally thin perfect conductor. The selection of an appropriate set of basis functions has also been well articulated in the literature [6]. Furthermore, it is well known that whenever a conductor is not perfect the currents will not become singular at the edge [3]–[5]. Since superconductors are the lowest loss conductors known, however, it is expected that such a set of basis functions will be a better approximation in this case than in any other real world situation. Except for distances within a penetration depth or so from the edge, the set of basis functions used in the above paper should accurately describe the currents in the superconducting film. This is one of the approximations made in the above paper for computational efficiency. However, it does not represent the only set of current basis functions [7] that may be used in conjunction with the complex resistive boundary condition.

In summary, the parameters in the form of an appropriate complex sheet resistance, quantifying the energy storage and energy dissipation in a superconductor, have been accounted for in the complex elements of the impedance dyadic via (44) and (45) of the above paper. The mechanics of the resulting numerical calculation are no different than if an infinitesimally thin perfect conductor were to be immersed in a slightly lossy layered medium. Since the impedance dyadic contains the description of energies stored and dissipated in the superconductor, the eigenvalue found by Galerkin's method is a complex quantity that corresponds to the complex propagation constant. The method of solution in the above paper specifically avoids the difficulties that are raised in the comments. It should be emphasized that the formulation, given by (44) and (45) in the above paper, is applicable in cases where the superconducting film is electromagnetically thin such that the resistive boundary condition, as given by (13) and (14) in the above paper, applies.

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Comments on "Conversions Between *S*, *Z*, *Y*, *h*, *ABCD*, and *T* Parameters which are Valid for Complex Source and Load Impedances"

Roger B. Marks and Dylan F. Williams

In his recent paper,[1] Frickey presents formulas for conversions between various network matrices. Four of these matrices (*Z*, *Y*, *h*, and *ABCD*) relate voltages and currents at the ports; the other two (*S* and *T*) relate wave quantities. These relationships depend on the definitions of the waves themselves in terms of voltage and current. Frickey's results are based on an unconventional definition of the waves, whose resulting properties are unfamiliar to most mircrowave engineers. As a result, application of his formulas can easily lead to catastrophic errors.

The scattering and transmission matrices of classical microwave circuit theory (e.g., [1]–[3]) relate the complex amplitudes of the counterpropagating traveling waves in a transmission line. These modal waves are solutions of Maxwell's equations whose dependence on the axial coordinate z is $e^{\pm\gamma z}$, where γ is the propagation constant. Ratios of the traveling wave amplitudes can be measured by classical slotted line techniques or with a network analyzer using a thru-reflect-line (TRL) calibration [4].

The classical circuit theory also allows the possibility of renormalizing the traveling waves by introducing a reference impedance Z_{ref} that may differ from the characteristic impedance Z_o. The resulting quantities form the basis of a renormalized scattering matrix. For instance, the renormalized reflection coefficient (one-port scattering matrix) Γ of a load of impedance Z_{load}, using a reference impedance Z_{ref}, is simply

$$\Gamma = \frac{Z_{\mathrm{load}} - Z_{\mathrm{ref}}}{Z_{\mathrm{load}} + Z_{\mathrm{ref}}} = \frac{Z_{\mathrm{load}}/Z_{\mathrm{ref}} - 1}{Z_{\mathrm{load}}/Z_{\mathrm{ref}} + 1}. \tag{1}$$

This familiar form is the basis of the Smith Chart, which provides a convenient graphical method of transforming between the reflection coefficient and the normalized load impedance $Z_{\mathrm{load}}/Z_{\mathrm{ref}}$, which, as shown by (1), uniquely determines Γ.

Instead of traveling waves, Frickey [1] makes use of parameters that Youla [5] defines and calls "waves"; a form of these parameters known as "power waves" has previously been applied to microwave circuits [6]. In spite of the terminology, Youla's parameters have little in common with waves. For instance, they do not depend exponentially or even monotonically on z [4]. Furthermore, the properties of Youla's parameters differ fundamentally from those of the renormalized traveling waves. For example, Youla's reflection coefficient $\hat{\Gamma}$ is

$$\hat{\Gamma} = \frac{Z_{\mathrm{load}} - Z^*_{\mathrm{ref}}}{Z_{\mathrm{load}} + Z_{\mathrm{ref}}} = \frac{Z_{\mathrm{load}}/Z_{\mathrm{ref}} - Z^*_{\mathrm{ref}}/Z_{\mathrm{ref}}}{Z_{\mathrm{load}}/Z_{\mathrm{ref}} + 1}. \tag{2}$$

Since (1) does not apply, the Smith Chart is *inapplicable* to Youla's parameters. In fact, $\hat{\Gamma}$ is not even uniquely determined by $Z_{\mathrm{load}}/Z_{\mathrm{ref}}$, as is Γ. As an illustration, the renormalized reflection coefficient of a short circuit ($Z_{\mathrm{load}} = 0$) is always $\Gamma = -1$, regardless of reference impedance Z_{ref}. In contrast, (2) shows that Youla's reflection coefficient of a short is equal *not* to -1 but to $-Z^*_{\mathrm{ref}}/Z_{\mathrm{ref}}$, which has magnitude 1 but is not generally real.

No microwave instrumentation or calibration known to us measures Youla's waves [4]. Thus, the equations of the above paper cannot be used to determine impedance parameters from measured scattering parameters. To illustrate, we used the multiline TRL calibration [7] to measure the scattering parameters of a small lumped resistor (with measured dc resistance $R_{\mathrm{dc}} = 59.3\ \Omega$) embedded in a coplanar waveguide. We measured the characteristic impedance Z_o of the transmission line using the technique of [8] and [9]. In applying (1) and (2), we made use of the fact that $Z_{\mathrm{ref}} = Z_o$, a condition which, as is well known, is mandated by the TRL calibration [4], [10]. We determined the resistor impedance Z_{load} first using (1). The result, shown in the solid curves of Fig. 1, closely tracks the resistor's anticipated behavior: the real part is approximately 59 Ω, and the imaginary part is small, approaching zero approximately linearly at low frequencies. When we instead used (2) to calculate Z_{load}, under the assumption that the measured reflection coefficient is actually $\hat{\Gamma}$, we found an anomalous result (dashed curves of Fig. 1).

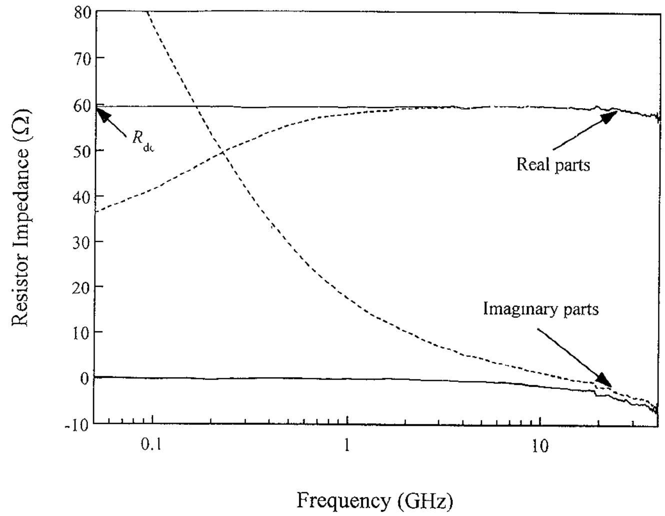

Fig. 1. The impedance of a small lumped resistor calculated, using $Z_{\mathrm{ref}} = Z_o$, from scattering parameters measured by the multiline TRL calibration. The solid curves are calculated from (1), the dashed curves from (2).

Due to the unconventional definition of Youla's waves, they can easily lead to erroneous results. For example, consider the simple flow graph of Fig. 2. When the two devices are joined at a reflectionless connector, we generally assume that, as long as the reference impedances at adjoining ports are identical, we can model the circuit by using the simple boundary conditions

$$b_3 = a_2 \tag{3}$$

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